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A 3D landslide analyses with constant mechanical parameters compared with the results of a probabilistic approach assuming selected heterogeneities at different spatial scales

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ABSTRACT: The aim of this paper is to compare the results of numerical analyses, related to an actual landslides (Roccamontepiano, Abruzzo, Italy), carried out assuming spatially constant mechanical parameters, with the correspondent results obtained including some selected heterogeneities at different scales. By laboratory tests, the values range of some important parameters of the soil materials, characterizing the selected situ and related to the assumed constitutive models, have been measured and evaluated. Then, FLAC-3D numerical analysis, assuming the averaged values of the parameters in each physical points of the geological system, has been elaborated. Parameters variability has been included in a second calculation, by the means of a method already proposed in literature, degrading the averaged numerical values in some way proportional to the standard deviations of the experimental measures obtained in laboratory. In all the previous calculations, however, spatial heterogeneities have been excluded. A large scale parameters variation, in particular with the depth, has been assumed. Then, a further analysis has been discussed considering the values of the parameters distributed randomly through the spatial grid, to take into consideration local variabilities and small scales. Finally the previous results have been compared with which has been obtained applying a probabilistic procedure, already proposed and discussed by the Authors in previous papers, carried out making several runs of the same numerical models, varying each time, in an automatic way, only the numerical realization of the parameters values whose ensemble was assumed belonging to a selected statistic. The comparison highlighted, for this kind of analyses, not only the obvious importance to include large variabilities, but also the less obvious importance of local inhomogeneities, in this case, around a deterministic trend of the mechanical parameters.

Analisi tridimensionali di frane assumendo parametri meccanici costanti, confrontate con i risultati conseguiti mediante l'applicazione di un Approccio Probabilistico, includendo eterogeneità a diverse scale spaziali

RIASSUNTO: Lo scopo del lavoro è il confronto tra i risultati di numerose analisi numeriche, eseguite su una reale frana situata presso Roccamontepiano (provincia di Chieti - Abruzzo), assumendo parametri meccanici spazialmente costanti o variabili in maniera stocastica, in considerazione della presenza di eterogeneità a differenti scale. Attraverso prove di laboratorio è stato possibile individuare il "range" di variazione dei valori dei principali parametri geotecnici dei terreni che caratterizzano il sito selezionato e che costituiscono le grandezze utilizzate nel modello costitutivo ritenuto più opportuno. Quindi, è stata eseguita un'analisi numerica con il codice di calcolo *FLAC_3D* assegnando i valori mediati dei parametri in ciascun nodo fisico del sistema geologico. In una seconda analisi è stata inclusa la variabilità stocastica dei parametri degradando i valori numerici medi in modo proporzionale ai valori di deviazione standard ottenuti in laboratorio. In tutte le precedenti analisi, comunque, le eterogeneità spaziali sono state escluse. Pertanto, successivamente, è stata effettuata una simulazione mediante l'inclusione della variabilità a grande scala dei parametri con la profondità. Quindi, è stata condotta un'ulteriore simulazione, considerando i valori dei parametri distribuiti in modo random, senza alcun trend deterministico, allo scopo di includere solo variabilità a piccola scala. Infine le analisi precedenti sono state confrontate con i risultati conseguiti mediante l'applicazione di un *Approccio Probabilistico*, proposto dagli Autori, svolgendo numerose simulazioni dello stesso modello numerico, variando ciascuna volta, tuttavia, in modo automatico, unicamente le realizzazioni numeriche dei parametri meccanici, il cui insieme si è assunto appartenere ad una specifica statistica. Il confronto ha evidenziato non solo l'ovvia importanza di includere variabilità a *grande scala*, ma anche la meno ovvia importanza delle disomogeneità locali a *piccola scala*, attorno, in questo caso, ad un trend deterministico dei parametri, variabile con la profondità.

Key terms: landslide; 3D numerical analyses; probabilistic approach
Termini chiave: instabilità dei versanti; analisi numeriche 3D; approccio probabilistico

1. Introduction

The study of landslides implies first of all a suitable characterization of the system by different point of views. One is about a reasonable reconstruction of local and global geometry of the involved geological structures of the system. Another one, regards the problem of how to handle, in a satisfactory way, the spatial variability of important parameters related to the selected constitutive models. Each modelling requires the selection of the most important features which can influence, more than others, the results of computer codes elaboration. To this purpose, the numerical experiments discussed in this paper have been applied to an actual landslide located in central Italy. In particular we studied the cinematic evolution of a travertine plate on a clay substratum, characterizing the territory of Roccamontepiano (Abruzzo Region). The geology has been characterized by both geo-electric and stratigraphyc surveys. The fractures lineations of the travertine, evidenced by aerophotograms, have not been considered, since, for the purpose of this paper, only some selected heterogeneities have been included, without invalidating anyway, the general achieved conclusions. Some samples laboratory tests have been carried out to define physical and mechanical parameters of lithotypes and their numerical variability. After an as accurate as possible reconstruction of the system, landslide modeling has been performed applying a three dimensional computer code: *FLAC-3D*. The soil was simulated by the code default *elastic-perfectly-plastic* model with failure being described by a composite *Mohr-Coulomb criterion* with a tension cut-off. Then the stability analyses have been performed considering the non convergence of the *maximum unbalanced force*, in order to avoid, by a computer time point of view, the cumbersome strategy implemented by default in FLAC-3D. The first approach we followed was the usually selection of averaged values of the most important mechanical parameters, different for each materials, but not spatially variable. Then to take into consideration the numerical variabilities of the parameters measured in laboratory, due to both to experimental errors and material heterogeneities, the averaged values have been slightly degraded as an already proposed strategy (Cherubini & Orr, 1999), but again without any spatial variability through a region characterized by the same material. Thus we selected a first simple kind of large scale heterogeneity: a linear parameters variation with the depth. Then we considered, for each material, a smaller scale variability alone, through a random Gaussian realization of the mechanical properties without any deterministic trend. Thus we also had to analyze the combined effect of the selected kind of variability at different spatial scales. To this purpose we discussed the comparison of the results carried out with the just described methodologies with the results obtained through a probabilistic approach, applied to the same landslide, proposed in a previous paper (Pasculli et al. 2006), based on fifty three runnings, considering each time a different value of some selected parameters, extracted randomly among the "ensemble" of all the possible "realizations" of the statistic, assumed to be Gaussian, to which the set of the numerical values of the considered parameters have been supposed to belong. The proposed probabilistic approach is briefly described in the following paragraphs. The comparison showed the importance to include in a landslide modeling, at least of the type selected in this paper, not only large spatial scale variability, but local parameters variation, as well, determined by a probabilistic approach, since, in this paper we have showed that the stability depends, also, on the particular statistical spatial realization, of the mechanical parameters.

2. Geological and Physical model reconstruction

The area selected just in order to apply the methodologies discussed in this paper, is localized at the border between two little villages, Roccamontepiano and Serramonacesca, on the eastern piedmont strip of the Maiella massif in Abruzzo (Italy). The main characteristic of this zone (Crescenti et al., 1987), called Montepiano, is the presence of a travertine plateau, which dominates both topographically and morphologically the sequence of the hilly landforms situated below. The plateau shape is sub rectangular and extends itself towards the Apennine direction. Its maximum length is about 2.3 km (along NW-SE direction) and its maximum width is about 0.7 km. Its altitude ranges from 610 up to 650 meters about the sea, slightly inclined towards NE. The edges of this travertine plate have been subjecting to a widespread landslides activities. In the past, it caused particularly dangerous events, as that one occurred on June 1765 which destroyed the whole built up areas. As a consequence, the relief is surrounded, almost completely, by detrital bodies, some ones very big, which characterize the most area of the south territory of Roccamontepiano village. The eastern sectors are interested by smooth hilly relief and are constituted essentially by clayey sandy Plio-Pleistocene sediments (see the Geological map, 1:5000 scale, reported in Fig. 1).

In order to specify the main lithological units of the entire area, the first step was a detailed geologic survey. The selected area shows some continental deposits which are characterized also by important geometry and thickness variability. So it was necessary to carry out some geophysical surveys. In particular vertical geo electrical methodologies have been applied, realized through different inter-electrodes distances in order to follow the contact, localized at different depths, between continental formation and the clay below. This kind of survey has been calibrated with continuum logs realized in the same area.

Overlapping both the data of the first and the second kind of surveying, an enough satisfactory physical model, in particular of the travertine plate, was obtained. The fractures lineation of the travertine, evidenced essentially by aerophotograms, have not been considered, since, for the purpose of this paper, only some selected heterogeneities have been included, without invalidating the general achieved conclusions. In consideration of the clayey nature of the substratum, as approximation, hydrogeology has not been included as a first approximation. Finally, laboratory tests have been performed to define some of the most important mechanical parameters and their variability ("min and max laboratory measure" in Table 1).

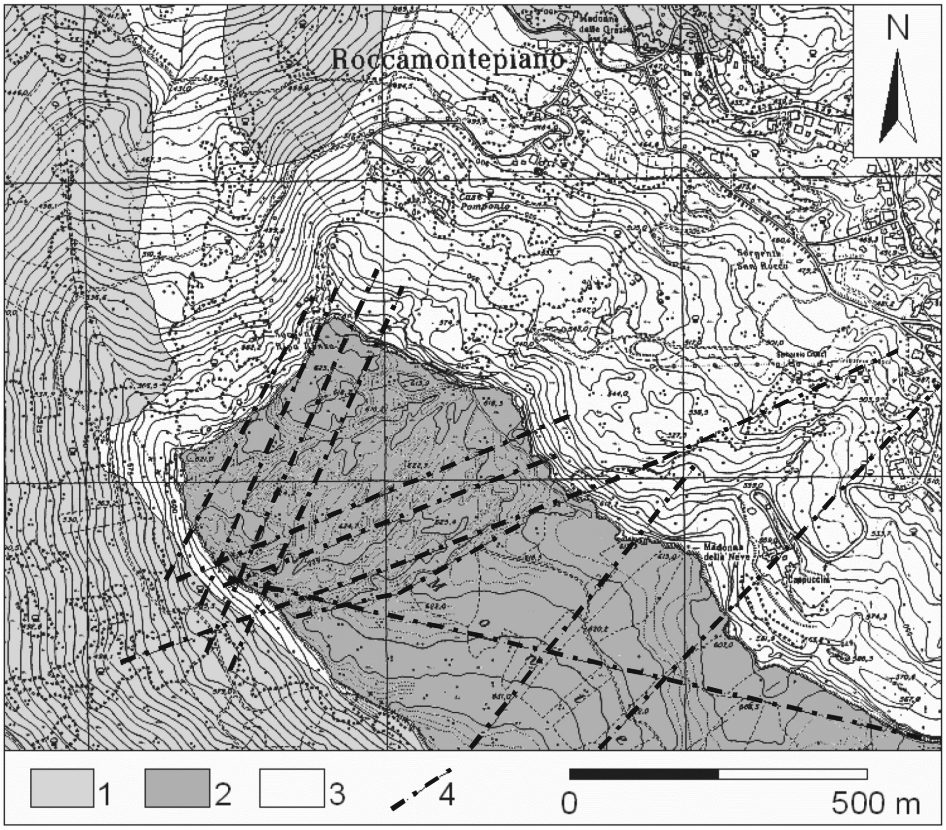

Figure 1. Geological map of Montepiano's area: 1) clayey lime; 2) Travertine; 3) landslide detritus; 4) fractures
Schema geologico dell'area di Montepiano: 1) limo-argilloso; 2) traveritno; 3) detrito di frana; 4) fratture

Table 1. Assumed values of selected physical-mechanical parameters
Valori assunti dei parametri fisico-meccanici

Mechanical parameter	Travertine				Clay			
	min. laboratory tests	max. laborator y tests	Case A $\frac{min+max}{2}$	Case B $x_k = x_m\left(1-\frac{\sigma_x}{2x_m}\right)$	min. laboratory tests	max. laborator y tests	Case A $\frac{min+max}{2}$	Case B $x_k = x_m\left(1-\frac{\sigma_x}{2x_m}\right)$
Unit weight (γ)(kN/m^3)	20.41	24.49	22.45	22.19	20.41	21.43	20.92	20.85
Friction Angle (φ)(^{o})	35.	45.	40.	39.38	23.	24.	23.5	23.43
Young Mod. (E) (GPa)	4.	10.	7.	6.625	0.05	0.36	0.21	0.191
Cohesion (c) (kPa)	0.	200.	100.	87.5	10.	35.	23.	21.5

3. Flac-3D model reconstruction and stability analyses

A particular care has been requested to build up the 3D numerical modeling of the selected area through the topographical map, completely digitized by a fine kriging interpolation (3x3 meters square meshes), the geological map and the geophysical surveys. Given the non small model dimension and the lithological complexity, just only two different kind of material have been assumed: travertine, which included all the related lithotypes (debris, travertine sands and so on, also characterized by degraded parameters) and clay with its surface covering. The further step was the reconstruction, in particular by geo electrical methods, of the contact between travertine and clay. Only the northern sector of the relief area has been modeled since stability problems are, in particular, localized in these zones. The related volume, selected in order to carry out numerical experiments, was 1400 meters wide, 1000 meters long and about 250-450 meters high. Numerical grid has been built up by more than 47,000 tetrahedrons whose largest dimension was of 10 meters at the surface, up to 40 meters in depth. The soil was simulated with the *elastic-perfectly-plastic* model with failure being described by a composite *Mohr-Coulomb criterion* with a *tension cut-off*, implemented by default in FLAC-3D (2000). Then, in order to make some further quantitative consideration about the risk of local instabilities relative to a relief like the geological system we are concerned with, we adopted the criteria to analyze how far the local stress state was from the failure region in the principal stress plane (σ_1, σ_3). Since the following *Mohr-Coulomb* with *tensile cut-off* failure criteria has been assumed (with the evidence of the symbols):

$$f^s = \sigma_1 - \sigma_3 N_\varphi + 2c\sqrt{N_\varphi}; \qquad f^t = \sigma_3 - \sigma^t$$
$$N_\varphi = [1+sin(\varphi)]/[1-sin(\varphi)]; \quad \sigma^t_{max}=c/tan(\varphi) \tag{1}$$

then the simplest indicative parameter which may be introduced for this purpose is just the distance d_i of the point P from the Mohr Coulomb line on the stress plane, analytically:(see Fig.2):

$$d_i = \frac{\left|\sigma_{1i} - \sigma_{3i} N_{\varphi\, i} + 2c_i\sqrt{N_{\varphi\, i}}\right|}{\sqrt{1+N^2_{\varphi\, i}}} \tag{2}$$

We called d_i the *strength failure indicator (SFI)*, relative to the *i-th* spatial mesh to which, in the probabilistic approach described in following paragraphs, peculiar parameters values are associated.

Finally, as stability analysis criterion, to avoid the expensive, by a consuming computer time point of view, methodology adopted as default by FLAC-3D, we followed the *strength reduction factor (SRF)* technique (Matsui & Sun, 1992; Zienkiewicz & Taylor, 1989). By means of this approach, the system was assumed to be in incipient collapse state as soon as the *maximum unbalanced force (MUF)* was going to diverge, as it is shown in Fig. 3.

4. Modeling with spatially constant parameters

For the first numerical experiment, which we have called case A, the averaged values of the mechanical parameters, also reported in Table 1, have been assumed constant through regions characterized by a single equivalent material. Fig. 4 shows both the displacement (left) and the *strength failure indicator (SFI)* (right) distributions on the surface of the selected geological system, provided by the numerical simulation at just ongoing instability condition, in this case, reached for a *strength reduction factor SRF*=1.50. It is worthy to note that points for which $d_i = 0$ have to be considered in plasticity condition. Below, in the same figure, we have included the display of material already in plastic state ($d_i = 0$: dark), while the other points have been reported in grey. The maximum calculated displacement was about 0.23 m. The zones actually interested by instabilities phenomena have been labelled by the ellipses A, B and C. As it is evident from the figure, landslides occurrences seem to be poorly estimated. In Fig. 5 density distribution (above) and displacement (below) at ongoing instability condition, along the section A-A' as indicated in Fig. 4, have been displayed. Plasticity condition has been affected some internal points as well, as it is shown in Fig. 5.

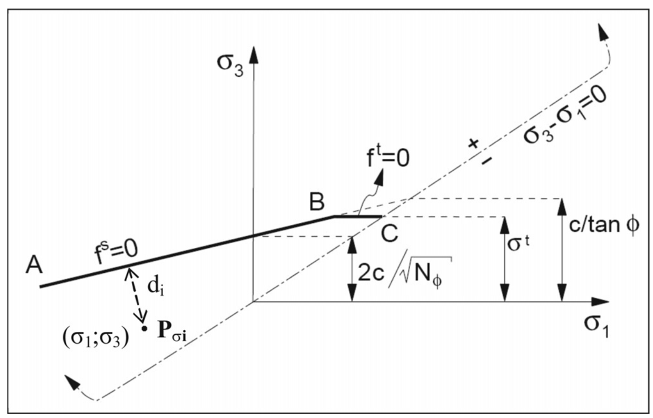

Figure 2. Principal stress planes with failure criteria associated to the *i-th* spatial mesh
Piani principali di sforzo con l'associato criterio di resistenza per la iesima mesh spaziale

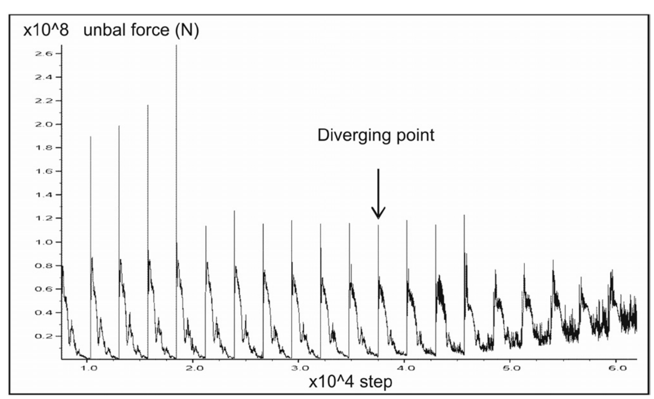

Figure 3. Max unbalanced force iterations
Iterazioni della massima forza sbilanciata

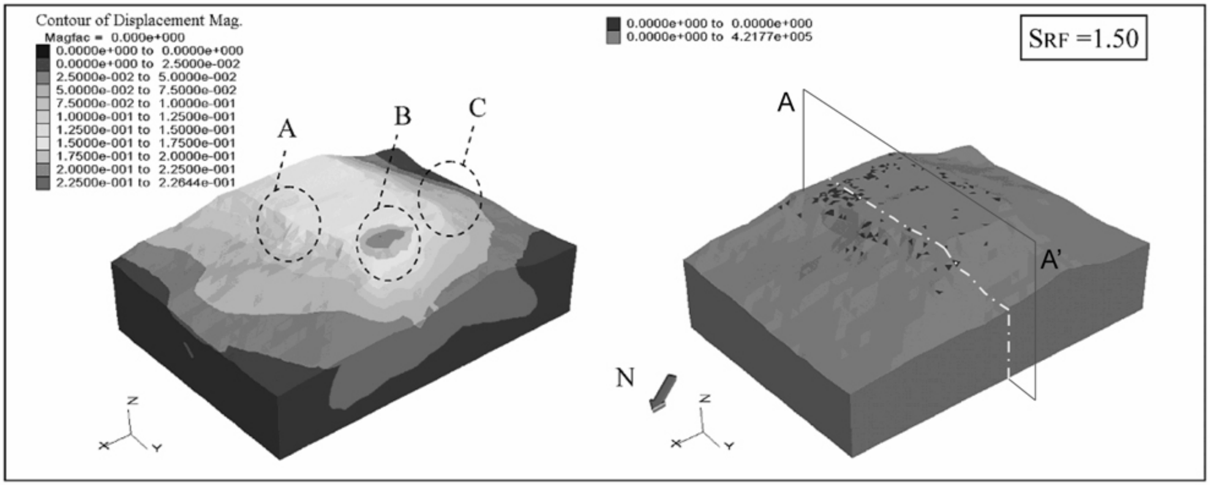

Figure 4. Case A; displacement and plastic points plots realized with constant average mechanical parameters
Caso A; spostamenti e localizzazione dei punti plasticizzati ottenuti utilizzando parametri meccanici medi

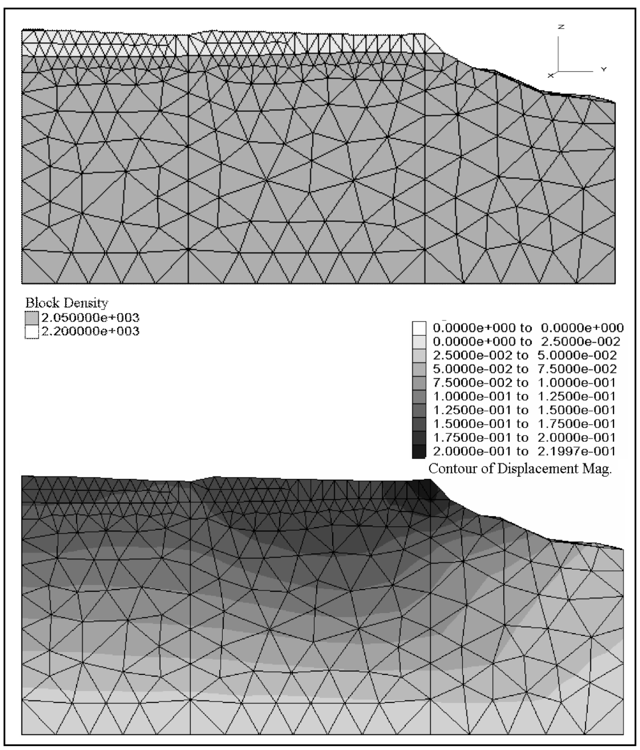

Figure 5. Section A-A': Case A, density and displacements distribution
Sezione A-A': Caso A, distribuzione della densità e degli spostamenti

The second numerical experiment, called case B, has been carried out assuming degraded averaged values of the parameters, to take into consideration variability, by means of the *characteristic value* introduced by EUROCODE 7, which may be evaluated by the following expression (after Cherubini & Orr, 1999):

$$x_k = x_m \left(1 - \frac{\sigma_x}{2x_m}\right) \qquad (3)$$

where x_k is the selected parameter, x_m the arithmetic average of the statistical ensemble to which x_k belongs, if experimental measures are provided, or it is supposed to belong, if the statistical distribution is just assumed, and $\sigma_x = \sqrt{\left(\sum_{i=1}^{N}(x_{ki} - x_m)^2\right)/N}$ is the standard deviation of the ensemble. For all the analyses discussed in the following, we have supposed, for simplicity and for the aim of this paper, that the numerical variability of a selected parameter: $x_{min} \le x_k \le x_{max}$, where x_{min} and x_{max} have been provided by the "min and max laboratory measure" values reported in Table 1, was completely related to spatial heterogeneities. Further, considering x_k as a stochastic variable, it is well known that, for the *Chebyshev inequality*, about 93% of the numerical values of x_k lies in the range: $x_{mean} - 4\sigma_x \le x_k \le x_{mean} + 4\sigma_x$ independently of the probability distribution function. Thus, in order to satisfy the double constraints $x_{min} \equiv x_{mean} - 4\sigma_x$ and $x_{max} \equiv x_{mean} + 4\sigma_x$, where $x_{mean} = (x_{min} + x_{max})/2$, we assumed:

$$\sigma_x = \frac{x_{max} - x_{mean}}{4} \qquad (4)$$

which satisfies directly the upper bound, while for the lower bound, introducing the relation (4) directly into the $x_{min} \equiv x_{mean} - 4\sigma_x$, it follows the requested identity:

$x_{min} \equiv x_{mean} - 4\frac{(x_{max} - x_{mean})}{4} \equiv 2x_{mean} - x_{max} \equiv x_{min}$. So the values related to case B in the Table 1 have been provided by relations (3) and (4).

Fig. 6 shows, again, both the displacement (left) and *SFI* distributions (right), with plasticity zone evidenced by dark colour, on the surface of the selected geological system, for a *strength reduction factor SRF*=1.45. The maximum calculated displacement was in this case about 0.28 m. As it should be expected, *SRF* of the Case B was lower than *SRF* of the Case A, while the opposite occurred for the maximum calculated displacement. Notwithstanding the numerically predicted size of the plasticity zone was more close to the actual situation than the Case A, also case B seemed to underestimate the landslides distribution intensity.

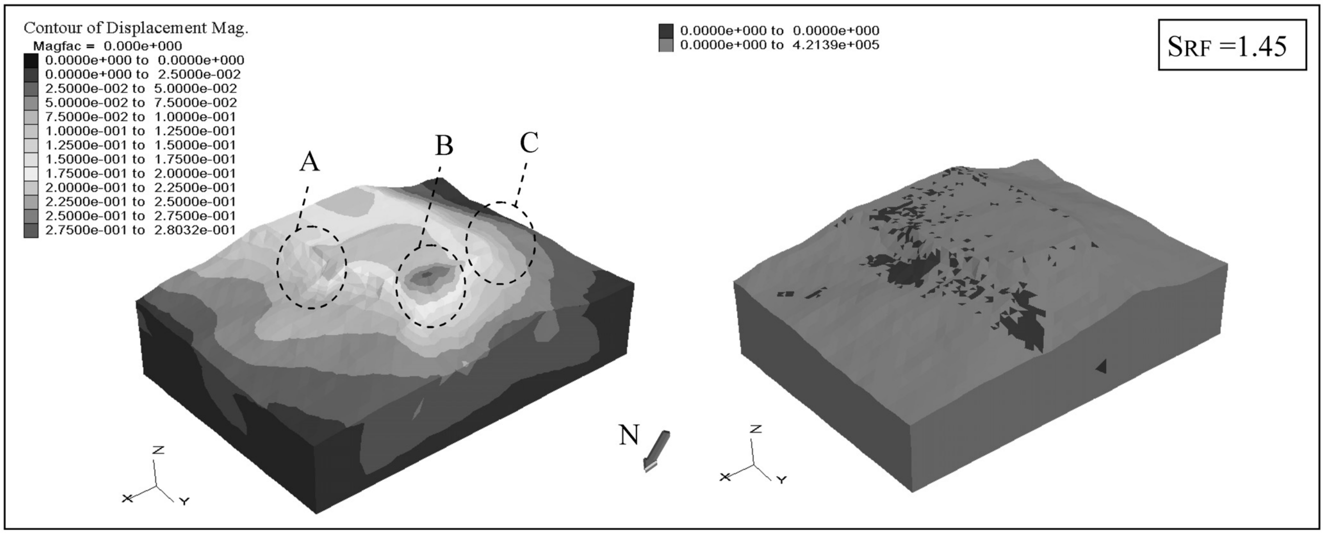

Figure 6. Case B; displacement and plastic points plots realized with degraded constant average mechanical parameters
Caso B; spostamenti e localizzazione dei punti plasticizzati ottenuti utilizzando parametri meccanici medi degradati

5. Modelling with heterogeneities at different spatial scale

Two kind of heterogeneities have been selected: the first one at a large spatial scale, whose analysis we have called Case C, included just only a linear clay density increase with the depth, while the travertine density value was supposed equal to that assumed for Case A and the second one, Case D, included just small scale variability due to local non homogeneities of the density in both travertine and clay region.

Thus for Case C we have assumed the following relation for the clay density:

$$\gamma_{clay}(P) = \frac{(\gamma_{max} - \gamma_{min})}{[h_{max} - h_{min}]} z(x,y) + \gamma_{min} \qquad (5)$$

$h_{max} = 650m$ and $h_{min} = 350m$ were, respectively, the maximum altitude and the lowest altitude, in the depth, we have assumed for all the simulations, while $0 \le z(x,y) \le [h_{max} - h_{min}]$ was the depth variable. The γ_{min} and γ_{max} values have been extracted from Table 1. Thus, on the surface $z(x,y) = 0$ it follows: $\gamma_{clay}(P) = \gamma_{min} \equiv 20.408 \; kN/m^3$, while on the plane $z(x,y) = [h_{max} - h_{min}]$ at the maximum depth considered for all the simulations: $\gamma_{clay}(P) = \gamma_{max} \equiv 21.429 \; kN/m^3$. All the other parameters x, for clay materials, have been assumed depending linearly on the density, as the following relation:

$$x = (x_{max} - x_{min}) \frac{[\gamma_c(x,y) - \gamma_{min}]}{(\gamma_{max} - \gamma_{min})} + x_{min} \qquad (6)$$

Fig. 7 displays the results of the case C simulation. The plots show a prediction of the instabilities distribution which was more satisfactory than all previous simulations, with a maximum displacement value of about 0.74 m. However *SRF*=1.60 was the highest numerically predicted value. Further, with a high *SRF* value, the *numerical safety* of the system may be excessively high. In Fig. 8, the section A-A' density and displacements are reported.

The case D was related to small scale variability of the density in both travertine and clay region. Thus, the parameters have been selected and implemented assuming a Gaussian distribution around x_{mean} with a standard deviation σ_x calculated by Equation (4). The related *Standard Gaussian distribution* has been provided by the Box & Muller, M. E. (1958) algorithm:

$$G\_norm = \sqrt{[-2ln(y\_ran1)]} \cdot cos(2\pi \cdot y\_ran2) \qquad (7)$$

where *y_ran1* and *y_ran2* were two non correlated (pseudo) random variables uniformly distributed in the interval (0,1) and provided directly by the FISH function URAND. This simulation, whose plots have not been reported, provided

SRF=1.45 and a maximum displacement equal to 0.21 m. The predicted surface plasticity points distribution, compared to the actual phenomena, appears similar to that one of case B, with degraded values of parameters and, any way, not so good as in the case C, even if the *SRF* value related to case D (*SRF*=1.45) was lower than case C (*SRF*=1.60). So the results of case D were more conservative than the results obtained by the case C.

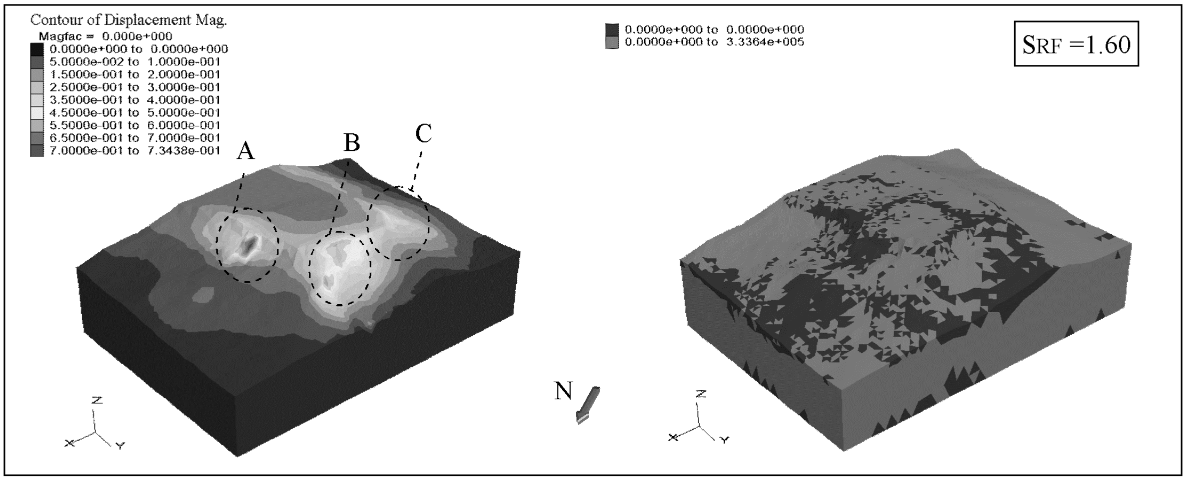

Figure 7. Case C; displacement and plastic points plots realized with clay linear parameters variation through the depth
Caso C; spostamenti e localizzazione dei punti plasticizzati ottenuti utilizzando nelle argille parametri meccanici variabili linearmente con la profondità

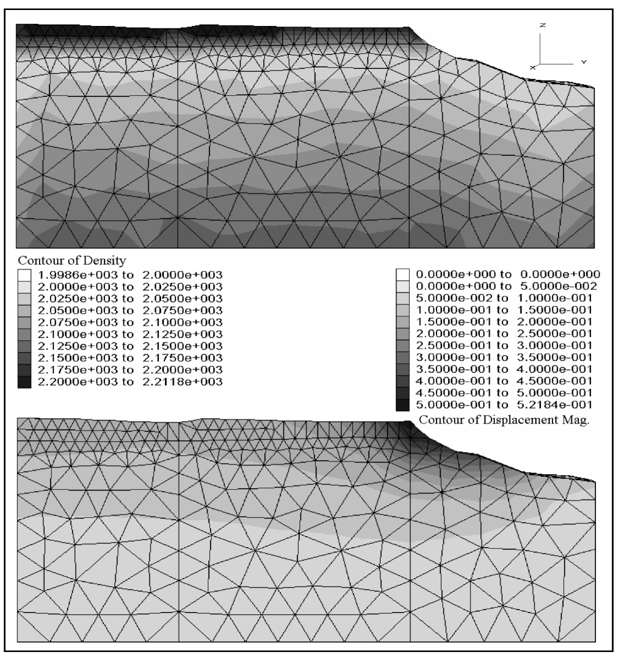

Figure 8. Section A-A': case C, density and displacements distribution
Sezione A-A': Caso C, distribuzione della densità e degli spostamenti

6. Probabilistic Approach

In a previous paper (Pasculli et al., 2006) a probabilistic approach has been proposed and the obtained results have been discussed. In order to make a comparison with the results so far analyzed, in the following a brief synthesis of the adopted method is reported. The usual procedure in numerical modelling implies the assignment of averaged values of mechanical parameters to each lithotypes layer. In a non linear condition it is debatable if this kind of approach may provide or it doesn't provide conservative results respect to actual values (Kaggwa, 2000). Thus in literature, in order to include the inevitable spatial variability, several methodologies have been discussed, but usually they are based on the following equation:

$$Phys(P) = \mu(P) + \delta \tag{8}$$

where $Phys(P)$ is the random parameter at the point *P*, $\mu(P)$ is the averaged value of the selected parameter and δ is the random perturbation usually assumed as a *Standard Gaussian*. In the proposed method, however, we adopted a simplified 3D version of a 2D approach, particularly suitable for granular soils (Pasculli & Sciarra 2002a,b) and implemented in FLAC-3D through the FISH program. In this model the selected parameters $Phys(P)$ are assumed to be the result of two physical causes: the first one is supposed to be due to the random formation of the granular deposit layer, while the second one is supposed to be due to a stochastic force, around a deterministic trend. The second term, which we call *stochastic-deterministic constraint (SDC)*, is supposed to include all the mechanical influence of the system on the soil element around the point *P*. The difference with the more common approach (Equation 8) is that the stochastic character of the phenomena is not imposed by a simple mathematical term (δ), but it is, in some way, justified by a physical point of view. Thus the following equation has been employed:

$$Phys(P) = Phys\_r(P) + [\mu(P) - Phys\_r(P)] \cdot [mean + \sigma \cdot G\_norm] \cdot e^{-[f(P)/s]^2} \quad (9)$$

where *Phys(P)* was again the random parameter (Young Modulus, Friction angle, cohesion, bulk unit weight and so on) at the point *P*; *Phys_r(P)* was the value due to the random formation of the selected soil layers; $\mu(P)$ was the deterministic trend of the *SDC*, while the term [$mean+\sigma \cdot G\_norm$] was a non dimensional number related to the random factor of the exerted *SDC*, in which "mean" was the average term, σ the standard deviation and *G_norm* the *Standard Gaussian distribution*, the last factor $e^{-[f(P)/s]^2}$ has been introduced to include weakness and perturbations (just like fractures, for example), localized along curves or geometrical zones described by the *f(P)/s* function. For all the numerical experiments discussed in this paper $e^{-[f(P)/s]^2} \equiv 1$ has been assumed. Further, the Gaussian random values of the selected parameters have been provided by the algorithm (7). The bulk unit weight of the materials has been selected as the random variable, while the other parameters have been employed as a linear function of it, in such a way as to save their variability through the numerical range, provided by laboratory tests and reported in the Table 1. For all the simulations, Poisson coefficient $\nu = 0.3$ has been assumed. For the travertine, the bulk unit weight has been supposed to be a Gaussian variable non depending on the depth. Thus from equation (9), setting $mean=0$ and $\sigma=0$, it followed: *Phys(P)=Phys_r(P)*. Further, the standard deviation was calculated by Equation (4). Random values eventually outside of the range, have been assumed to be equal to the mean. Then we employed the following relation: $\gamma_{ty} = \gamma_{mean} + (\sigma_\gamma \cdot G\_norm)$ on which the other parameters were linked through the expression (11). On the other hand, the unit weight of the clay material has been supposed to change linearly through the depth. So from Equation 9 and after the aforementioned discussion, the following expression for the clay density has been employed: $\gamma_c = \gamma_c\_r + [\mu - \gamma_c\_r] \cdot [1 + 0.125 \cdot G\_norm2]$, where $\gamma_c\_r = \gamma_{mean} + (\sigma_\gamma \cdot G\_norm1)$ was the Gaussian random value of the density around γ_{mean} with the standard deviation σ_γ, *G_norm1* and *G_norm2* were two *Standard Gaussian* variables extracted by two different, subsequent calls to the FISH (pseudo) random routine URAND.

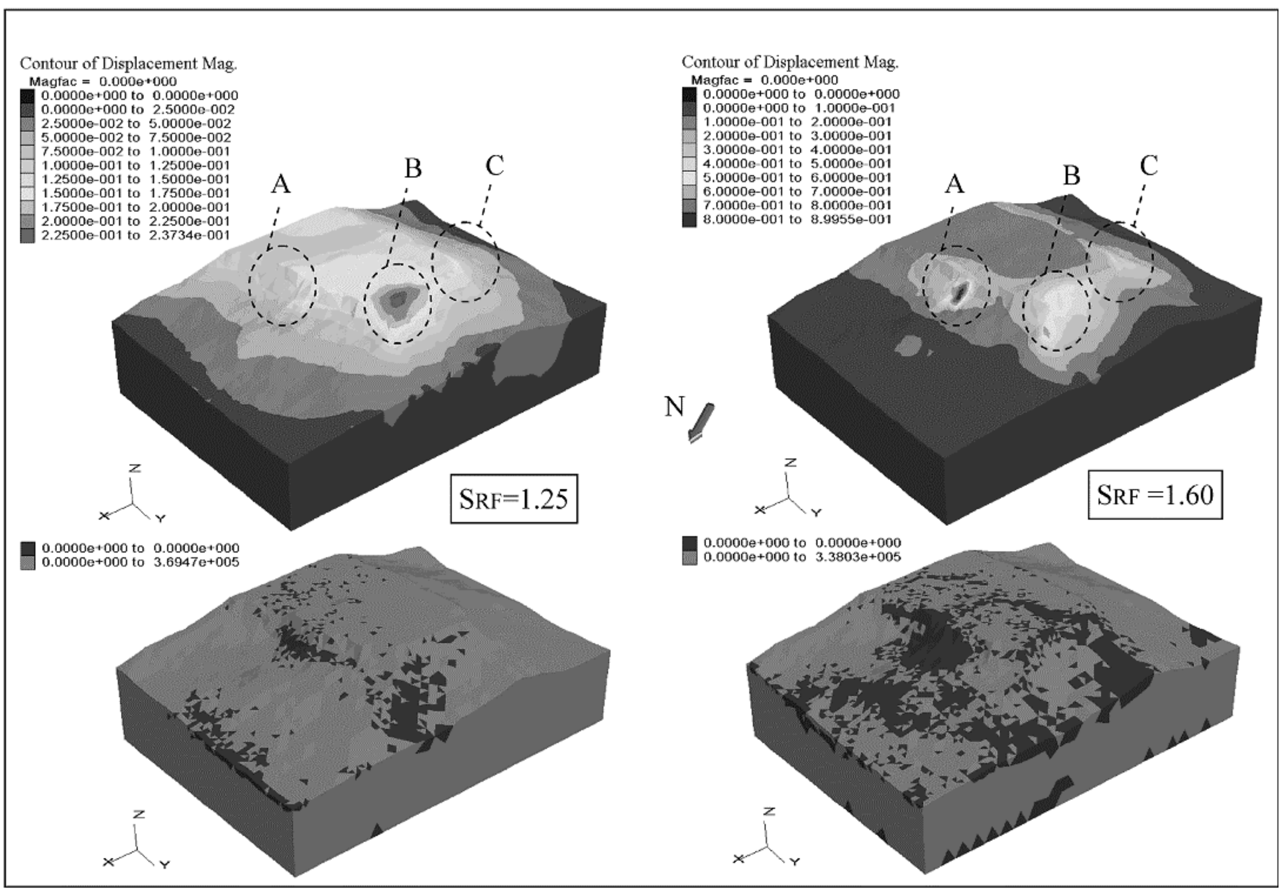

Figure 9. Displacements and plastic points plots for the two realized minimum and maximum *SRF* values
Spostamenti e localizzazione dei punti plasticizzati per i due valori minimo e massimo di SRF realizzati

The term μ was the deterministic trend of the effects of the total force exerted on the soil element, which, in this case, was supposed to be due to the lithostatic loading and assumed to be equal to Equation (5). The variation law of the other parameters has been supposed to follow linear trends (Eq. 6), similar to that followed by travertine's parameters. In the proposed approach, the mechanical parameters correlation between different soil points and their scale fluctuations (Fenton & Vanmarcke, 1990; Vanmarke, 1977) have not been included. This assumption is partially justified by the large scale of the whole system and by the necessity to carry out as fast (by computer time point of view) as possible simulations.

After having stated the basis of the proposed method, 53 running have been carried out, considering each time, in an automatic way, a different value of the selected parameters, extracted randomly among the "ensemble" of all the possible "realizations" of the aforementioned statistic, to which the set of the numerical values of the considered parameters has been supposed to belong.

Fig. 9 shows the view of two realizations related to the minimum and the maximum values of the *SRF* number, provided by the simulations, respectively equal to 1.25 and 1.6. It is worthwhile to observe that for the same value of the *SRF* number, both different spatial displacements and different plastic soil material distributions may be numerically expected as it was discussed in Pasculli (2006), in which for *SRF*=1.55 the max "numerical" displacements ranged from 0.55 m up to 0.74 m. It should be noted that by just varying randomly the realization of the mechanical parameter values and their physical point assignment, roughly around the same average trend in depth, a numerical modeling like that we have proposed, may predict, trough 53 runs, a max soil displacements ranging from 0.24 m up to 0.9 m, so with a factor of almost 4!7.

7. Results analyses, statistical considerations and comparison

In the histogram displayed in Fig. 10, we reported the *probabilistic distribution of the strength reduction factor*, defined as (Pasculli et al., 2006): "*the occurrence number of the analyses for which a specific value of the SRF (strength reduction factor) has been provided by the simulations and their frequencies related to the total number of runs*". In the figure we included, in black, the *SRF* values obtained by the cases A, B, C, and D already discussed in previous paragraphs.

Moreover, to analyze the risk of instability at an intermediate scale, we divided the selected geological system in six 50 meters thick sectors, as reported in Fig. 11, in an exploded view. Then, in the framework of a numerical "probabilistic micro zoning" procedure related to instability risk, we introduced two simple numerical indicators, calculated for each run, related to each spatial sectors in which we have divided the selected system. The first one was a plasticity ratio: $DI_{ik} = V_{ik} / V_{itot}$ where V_{ik} was the soil mesh volume of the sector *i-th* in a plasticity state, obtained in the *k-th* running, while V_{itot} was the total volume of the *i-th* sector layer. The second one, calculated again for each sector, was an averaged strength failure indicator, weighted by means of the "importance" ratio of the volume V_j of the *j-th* mesh over the total volume V_{itot} of the *i-th* sector:, $\mu_{di} = \frac{1}{N_i}\sum_{j=1}^{N_i}\left[d_j \frac{V_j}{V_{itot}}\right]$ where N_i was the number of the meshes in the *i-th* sector. The first indicator was related to the calculated mobilized mass for each sector and for each run, therefore to how much the soil material inside the selected sector has been damaged. So we called it the *numerical degrade indicator (DI)*. The second one specified, numerically, how far the stress state of the soil material was from failure conditions. We called it: *numerical hazard stress state indicator (HSSI)*. Then to analyze how much each single spatial mesh was close to its failure condition, the *strength failure indicator (SFI)*, was employed. The set of all *SFI* values determined, for each run, an ensemble with an arithmetic mean, characteristic for each of the six sectors in which the landslide area has been divided.

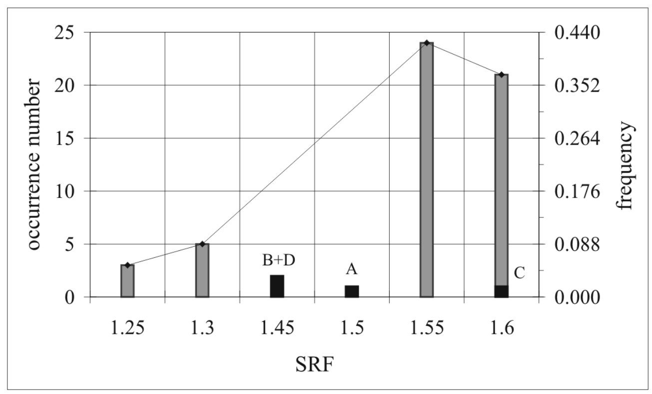

Figure 10. Probabilistic distribution of the strength reduction factor
Distribuzione probabilistica del fattore di riduzione della resistenza

A statistical analysis of the *SFI* parameter of each sector was discussed in order to introduce a *numerical landslide micro-zonation* concept. Thus, for each sector, we analyzed the linear correlation coefficient among the total plastic soil material, expressed by the degrade indicator *DI* and the calculated *SRF* number:

$$r_{DI,SRF}\sum_{j=1}^{53}(DI_j^t - \overline{DI}_j^t)(SRF_j - \overline{SRF}),$$

where DI_j^t and SRF_j were, respectively, the total plastic soil material for a specific sector and the strength reduction factor provided by the j-th run, while the others symbols were their average values and the related standard deviations. The coefficient $r_{DI,\,SRF}$, which have been sketched in Fig. 11 for each sector, shows, by a statistical point of view, how much the selected sector was involved in the total stability occurrence. By this analysis the sectors 4, 2, and 1 were the most involved ones. In Fig. 12 we reported the occurrence number of the analyses for which a specific ratio value of the plastic soil mass, over the total mass in the selected sector (*DI* value) has been provided by the 53 simulations. Thus it would be straightforward, normalizing the ordinate to the total simulation number, to obtain an estimation of the *Probability of the failed soil mass amount* in each sector.

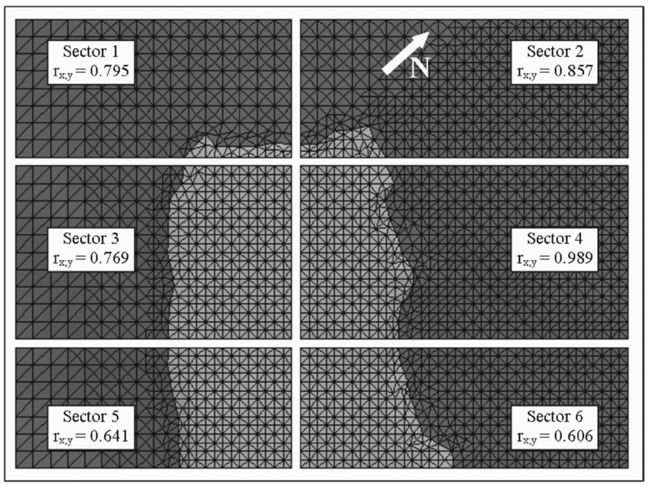

Figure 11. Analysed sectors map
Schema dei settori analizzati

The calculated percentage of the soil in a plastic state, ranged from 1.3% up to 66%. It is interesting to note that the effects of the selected random spatial variability have been more important for sectors 1, 2, 3, and 4, while the "probability distributions" related to sectors 5 and 6 have not been so spread out along the whole range, thus these sectors have been less sensitive to the inclusion of the variability of the mechanical parameters in the modelling. In the same figure we reported the calculated values for A, B, C and D cases discussed in this paper. It is interesting to note that the cases A, (constant parameters), B (degraded parameters) and D (only small scale heterogeneities) provided lower values of the volume percentage in plasticity condition for the sectors more involved in system instability, than the values obtained in the case C (large scale variability). Also it is worthy to note that linking both large and small scale variabilities, the upper bound of the spectrum of the failed materials percentage was higher than those ones provided by all the four cases, elaborated without the probabilistic approach.

Furthermore we analyzed another important parameter: the numerical *stress state hazard indicator*. Also in this case, the selected parameter μ_{di} has been calculated for each sector and for all the runnings. Due to its large numerical variability (several magnitude order), we considered its natural logarithm and then we plotted the related *occurrence number* in Fig.13.

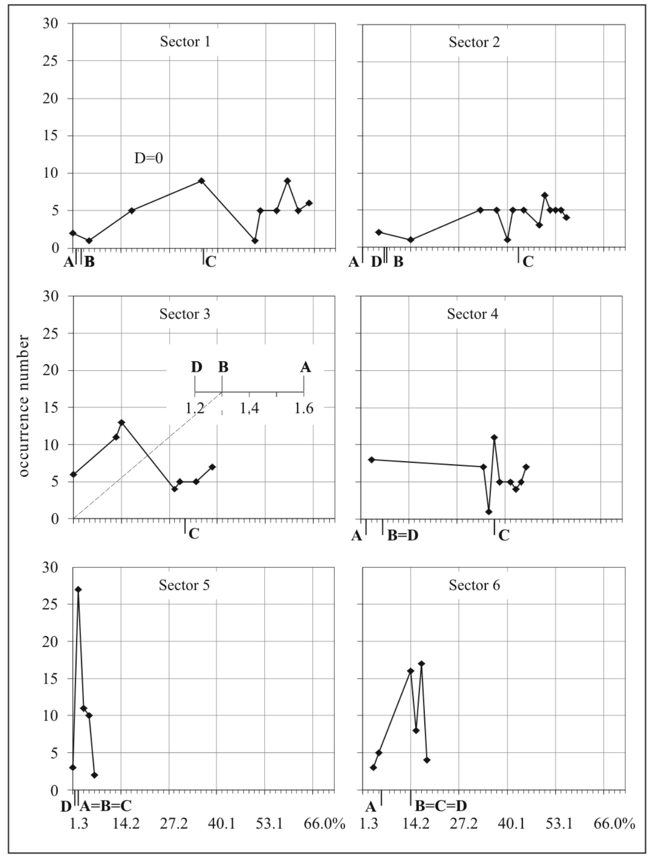

Figure 12. Occurrence number vs percentage of the failed soil mass amount
Numero di eventi per i quali si è ottenuta la stessa percentuale di massa elasticizzata

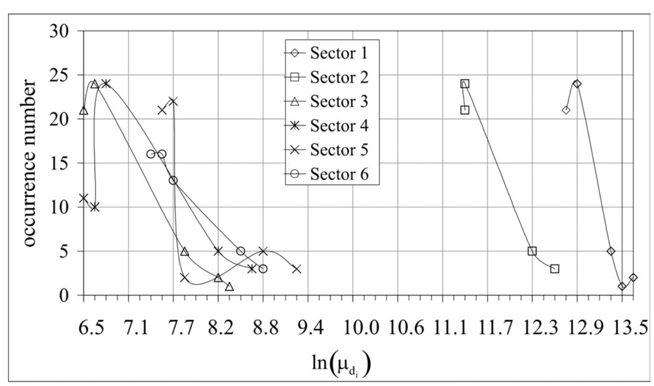

Figure 13. Occurrence number vs natural logarithm of the *stress* state hazard indicator
Numero di eventi per i quali si è ottenuto lo stesso logaritmo naturale dell'indicatore di pericolosità da stress

The parameter ranged from 0.7 kPa up to 730 kPa. Since sectors 3 and 4 show low values of the *stress distance* from the "plasticity region", they are also the regions at the highest hazard, by a numerical point of view, to be interested by successive local instability phenomena. On the other hand, in the same figure, it is interesting to note that sectors 1 and 2, which have already experienced landslide occurrences, show a lower hazard of further dangerous events, because the stress state related to the soil materials have already suffered a plasticity action with a consequent stress unloaded of the materials.

In Table 2 we have reported $ln(\mu_{di})$ values provided by the Probabilistic Approach (range and maximum Mode) and by the cases A, B, C and D. It is interesting to note that the values of the natural logarithm of the *stress state hazard indicator* predicted by case C (in bold) was more close to the *max mode value*, in bold (the value in correspondence of the maximum occurrence number in the *Probabilistic approach*), than the other ones. Thus it may be argued that in a *Probabilistic Approach* like that discussed in this paper as well, the selected large scale heterogeneities determine the response characteristics of many simulations (high *occurrence number*), but small scale heterogeneities linked with larger ones determine the tails response with a non vanishing *occurrence numbers* and, consequently, with non negligible *frequency number,* making their simulation as necessary.

Table 2 Stress state hazard indicator for each sector and for each adopted method typology
Indicatori di pericolosità dello stato di stress per ciascun settore e per ciascuna tipologia di metodo adottato

sector	1	2	3	4	5	6
$ln(\mu_{di})$ range	12.75-13.5	11.4-12.6	6.5-8.35	6.5-8.65	7.55-9.25	7.4-8.8
Max mode value	**12.90**	**11.42**	**6.65**	**6.80**	**7.70**	**7.40**
Case A	13.00	11.79	7.26	7.67	7.62	7.55
Case B	13.05	11.87	7.36	7.84	7.79	7.69
Case C	**12.64**	**11.18**	**6.61**	**6.64**	**7.55**	**7.44**
Case D	13.02	11.97	7.74	8.09	8.03	8.07

8. Conclusion

In this paper an actual landslide has been studied by usual methods, applying the three dimensional commercial computer code FLAC-3D. The results have been compared with the response of a *Probabilistic Approach*, already applied and discussed in previous papers of the Authors. For all the analyses, the soil has been simulated by the FLAC-3D default *elastic-perfectly-plastic* model with failure being described by a composite *Mohr-Coulomb criterion* with a tension cut-off. Then the stability analyses have been performed considering the non convergence of the *maximum unbalanced force*, in order to avoid, by a computer time point of view, the cumbersome strategy implemented by default in the selected code. The simulations have been carried out assuming, for each case, different values of the selected mechanical parameters, whose numerical ranges have been supplied by laboratory tests related to each materials, in order to take into consideration large and small heterogeneities. Thus four cases have been performed: case A, assuming the spatial constancy of the parameters in the travertine and in the clay, equal to their average values; case B, employing their average degraded values in order to include their variability, but also with the assumption of their spatial constancy; case C, assuming a linear variation of the clay parameters in the depth, with the aim to employ a large scale heterogeneities; case D, for each material, mechanical parameters have been assumed as random, without any spatial trend, in order to study the effects of small scale variability.

In the aforementioned *Probabilistic Approach*, a modeling based on a random number generation of numerical values of the selected parameters, has been applied through fifty three runs of the same model, but each time with a different spatial realization of the assumed Gaussian statistic. Thus, instead of being concerned with a single *strength reduction factor SRF*, as in the usual practice, the application of this method provided a statistical distribution of the numerical values, which SRF numbers may assume. Further, in order to introduce a *micro zoning* procedure, we discussed some other statistical parameters: the *degrade indicator (DI)* related to how much the soil material inside the selected sector has been damaged; the *stress state hazard indicator (SSHI)* which specifies how far the stress state of the soil material was and is from failure conditions. Through these indicators, the zones which, by a probabilistic point of view, may be affected by a high grade of damage or by a risk of damage have been localized.

The comparisons of the previously four cases with the response of the *Probabilistic Approach* have been evidenced that simulations with constant parameters (case A), degraded parameters (case B) and only small scale heterogeneities (case D) provided the *non conservative* lowest values of the *volume percentage of the failed materials* for the sectors more involved in system instability, while the numerical results of the case with large scale variability (case C) were more conservative, but, any way, lower than the upper bound values related to the responses supplied by the *Probabilistic Approach*, for which a large scale variability have been linked with a smaller one (local heterogeneities).

Furthermore the *stress state hazard indicator* predicted by case C (in bold) was more close to the *max mode value* (in bold), the value in correspondence of the maximum occurrence number in the *Probabilistic Approach*, than the other ones. Thus it may be argued that in a Probabilistic approach like that one discussed in this paper as well, the selected large scale heterogeneities determine the response characteristics of many simulations (high *occurrence number*), but small scale heterogeneities, linked with larger ones, determine the tails response with a non vanishing *occurrence numbers* and, consequently, with a non negligible *frequency number* of occurrence.

On the other hand, the *strength reduction factor SRF*, indicative of a whole scale safety, related to the case C, was the highest value provided by all the simulations, showing that local conservativism (high *volume percentage of the failed materials*) doesn't imply a whole scale conservativism. The immediate implication of the last observation is the utility to perform a *Probabilistic Approach* to obtain a spectrum of responses and probabilistic distributions instead of a unique result. Further, it is interesting to note that by just varying randomly the realization of the mechanical parameter values and their physical point assignment, roughly around the same average trend in depth, a numerical modeling like that we have discussed, may predict, trough 53 runnings, a max soil displacements ranging from 0.24 m up to 0.9 m, so with a factor of almost 4! Another point evidenced by the previous analyses was that each sectors of a system may be more or less affected by local variabilities. *This suggests, with the help of the other geological tools, a spatial optimization of eventually further surveys, excluding, partially, the zones which have been revealed less sensitive*.

Future improvements will imply the exploration of the effects of other statistics, the inclusion of points correlation, the inclusion of fluctuation scale, the provision of much more simulations in order to obtain a more suitable ensemble by a statistical point of view.

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